Multi-objective Portfolio Optimization Model

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Abstract

Multi-objective non-linear programs occur in various fields of applications in O.R. One of the applications of such program is portfolio selection problem. In this paper, we first consider a multi-objective Portfolio Selection based model and next added another entropy objective function, which is used by Shannon’s measure of entropy and then this problem is formulated in generalized form. Fuzzy programming technique is used to solve the problems. The models are illustrated with numerical examples.

Keywords and Phrases: Portfolio optimization, Multi-objective model, Entropy, Fuzzy mathematical programming.

1. Introduction

The theory of mean-variance efficient portfolios was first given by Markowitz [5] who also gave his critical line method for finding these. Markowitz published his work, which paved the foundation of the modern portfolio analysis. It combines probability and optimization theory to model the behavior of economic agents under uncertainty. Roll [6] gave an analytical method for finding modified mean-variance

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efficient portfolios in which short sales are allowed. Wang et. al.[7] presented single objective portfolio optimization model using fuzzy decision theory, possibilistic programming and interval programming. Inuiguchi and Tanino [2] proposed a new approach to the possibilistic portfolio selection problem. Usefulness of entropy optimization models in portfolio selection based problem are illustrated in two well-known books ([3],[4]). But discussion on entropy based multi-objective portfolio selection models is rarely presented in literature.

Suppose that a prosperous individual has an opportunity to invest an asset (i.e. a fixed amount of money) in n different bonds and stocks. Let \( x = (x_1, x_2, \cdots, x_n)^T \), where \( x_j \) is the proportion of his assets invested in the j-th security. The vector \( x \) is called portfolio. Clearly, a physically realizable portfolio must satisfy \( x_j \geq 0, \) \( (j=1, 2, \cdots, n) \), \( \sum_{j=1}^n x_j = 1 \). The agents are assumed to strike balance between maximizing the return and minimizing the risk of their investment decision. Return is quantified by the mean, and risk is characterized by the variance, of a portfolio assets. The return \( R_j \) for the j-th security, \( (j=1, 2, \cdots, n) \), is a random variable, with expected return \( r_j = E(R_j) \). Let \( R = (R_1, R_2, \cdots, R_n)^T \), \( r = (r_1, r_2, \cdots, r_n)^T \). The return for the portfolio is thus \( R^T x = \sum_{j=1}^n R_j x_j \) and expected return \( E_r(x) = E(R^T x) = \sum_{j=1}^n r_j x_j \).

Let \( \sigma_{ij} \) be the covariance matrix of a random vector \( R \), the variance of the portfolio is \( \sigma_r(x) = \text{Var}(R^T x) = \sum_{j=1}^n \sum_{i=1}^n \sigma_{ij} x_j x_i \) where

\[
\sigma_{ij} = \begin{cases} \sigma_j^2 & \text{when } i = j \\ \rho_{ij} \sigma_j \sigma_i & \text{when } i \neq j \end{cases}
\]

\( \sigma^2_j \) is the variance of \( R_j \) and \( \rho_{ij} (= \rho_{ji}) \) is the correlation coefficient between \( R_i \) and \( R_j \) for all \( i, j = 1, 2, \cdots, n \).

**Model- I: Portfolio Selection Problem (PSP)**

The two objectives of an investor are thus to maximize the expected value of return and minimize the variance subject to a constraint of a portfolio. So the Portfolio Selection Problem (PSP) is:

\[
\text{Maximize } E_r(x) = \sum_{j=1}^n r_j x_j , \quad (1.1)
\]
Minimize \( V_r(x) = \sum_{j=1}^{n} \sum_{i=1}^{n} \sigma_{ij} x_j x_i \),

subject to
\[ \sum_{j=1}^{n} x_j = 1, \]

and \( x_j \geq 0, \ j = 1, 2, \ldots, n. \)

Markowitz’s mean variance criterion simply states that an investor should always choose an efficient portfolio. In a certain number of portfolios with only maximum expected return security and with only minimum variance security are efficient portfolios, and the criterion does not give a way of choosing from them, but no investor would choose these because these are not sufficiently diversified. Therefore there is a scope for introducing another criterion viz one for diversification and the best candidate for this. It is to maximize the entropy objective function

\[ E_n(x) = -\sum_{j=1}^{n} x_j \log x_j \]

**Model- II: Portfolio Selection Problem with Diversification (PSPD)**

So real life problem in analogy to problem (1.1), is a Portfolio Selection Problem with Diversification (PSPD), which can be written as

Maximize \( E_n(x) = -\sum_{j=1}^{n} x_j \log x_j \), \quad (1.2)

Maximize \( E_r(x) = \sum_{j=1}^{n} r_j x_j \),

Minimize \( V_r(x) = \sum_{j=1}^{n} \sum_{i=1}^{n} \sigma_{ij} x_j x_i \),

subject to
\[ \sum_{j=1}^{n} x_j = 1, \]

and \( x_j \geq 0, \ j = 1, 2, \ldots, n. \)

**Model-III: Generalized Portfolio Selection Problem with Diversification (GPSPD)**

For generalization of the above model, an investor can construct a diversification portfolio based on m potential market scenarios from an investment universe of n
assets. Let $R^k_j$ (j=1,2,⋯,n, k=1,2,⋯,m) denote the return of the j-th asset and let
\[ R_k(x) = \sum_{j=1}^{n} R^k_j x_j \]
denote the portfolio return with expected return $E_{r_k}(x) = \sum_{j=1}^{n} r^k_j x_j$,
and $\sigma^k_{ij} = \begin{cases} (\sigma^k_j)^2 & \text{when } i = j \\ \rho^k_{ij} \sigma^k_j \sigma^k_i & \text{when } i \neq j \end{cases}$

where $(\sigma^k_j)^2$ is the variance of $R^k_j$ and $\rho^k_{ij} (= \rho^k_{ji})$ is the correlation coefficient between $R^k_j$ and $R^k_i$ (for all i, j =1, 2, ⋯, n) for the k-th market scenario at the end of investment period then $V_{r_k}(x) = \sum_{j=1}^{n} \sum_{i=1}^{n} \sigma^k_{ij} x_j x_i$, denote the risk for the k-th scenario.
So generalized Portfolio Selection Problem with Diversification is a multi-objective problem as follows:

Maximize $E_n(x) = -\sum_{j=1}^{n} x_j \log x_j$ \hspace{1cm} (1.3)
Maximize $E_{r_1}(x) = \sum_{j=1}^{n} r^1_j x_j$ ,
Maximize $E_{r_2}(x) = \sum_{j=1}^{n} r^2_j x_j$ ,
..............................,
Maximize $E_{r_m}(x) = \sum_{j=1}^{n} r^m_j x_j$ ,
Minimize $V_{r_1}(x) = \sum_{j=1}^{n} \sum_{i=1}^{n} \sigma^1_{ij} x_j x_i$ ,
Minimize $V_{r_2}(x) = \sum_{j=1}^{n} \sum_{i=1}^{n} \sigma^2_{ij} x_j x_i$ ,
..............................,
Minimize $V_{r_m}(x) = \sum_{j=1}^{n} \sum_{i=1}^{n} \sigma^m_{ij} x_j x_i$ ,
subject to
\[ \sum_{j=1}^{n} x_j = 1 \]
\[ x_j \geq 0, \ j=1, 2, \cdots, n. \]
2. Mathematical Analysis of Multi-objective Non-Linear Programming (MONLP) Problem and Its Solution Procedure

2.1. Multi-Objective Non-Linear Programming (MONLP) Problem

A general multi-objective non-linear programming (MONLP) problem may be taken in the following VMP:

Minimize k non-linear objective functions
\[ Z_1(x_1, x_2, x_3, \ldots, x_n), \]
\[ Z_2(x_1, x_2, x_3, \ldots, x_n), \]
\[ \ldots \ldots \ldots, \]
\[ Z_k(x_1, x_2, x_3, \ldots, x_n), \]
subject to the inequality constraints
\[ g_1(x_1, x_2, x_3, \ldots, x_n) \leq b_1, \]
\[ g_2(x_1, x_2, x_3, \ldots, x_n) \leq b_2, \]
\[ \ldots \ldots \ldots \ldots \]
\[ g_m(x_1, x_2, x_3, \ldots, x_n) \leq b_m, \]
and boundary restrictions
\[ l_i \leq x_i \leq u_i \quad (i=1, 2, \ldots, n). \]
i.e. in vector form
Minimize \( Z(x) = [Z_1(x), Z_2(x), \ldots, Z_k(x)] \)
subject to \( x \in X = \{x : g_j(x) \leq b_j \, (j = 1, 2, \ldots, m) \text{ and } l_i \leq x_i \leq u_i \, (i=1,2, \ldots, n)\}. \)

A direct application of the optimality for single-objective non-linear programming to the MONLP (2.1.1) leads us to the following complete optimality concept.

Definition 2.1.1. (Complete Optimal Solution)

\( x^* \) is said to be a complete optimal solution to the MONLP (2.1.1) if and only if there exists \( x^* \in X \) such that \( Z_r(x^*) \leq Z_r(x) \), for \( r = 1, 2, \ldots, k \) and for all \( x \in X \).

However, when the objective functions of the MONLP conflict with each other, a complete optimal solution does not always exist and hence the Pareto Optimality Concept arises and it is defined as follows.
Definition 2.1.2. (Pareto Optimal Solution)

\( x^* \) is said to be a Pareto Optimal solution to the MONLP (2.1.1) if and only if there does not exist another \( x \in X \) such that \( Z_r(x^*) \leq Z_r(x) \) for all \( r = 1, 2, \cdots, k \) and \( Z_j(x^*) \neq Z_j(x) \) for at least one \( j, j \in \{1, 2, \cdots, k\} \).

2.2. Fuzzy Programming Technique to Solve MONLP Problem

Zimmermann [9] showed that fuzzy programming technique could be used nicely to solve the multi-objective programming problem.

To solve VMP (2.1.1) problem, following steps are used:

Step 1: Solve the VMP (2.1.1) as a single objective non-linear programming problem using only one objective at a time and ignoring the others. These solutions are known as ideal solution.

Step 2: From the results of Step 1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each ideal solution, payoff matrix can be formulated as follows:

\[
\begin{bmatrix}
Z_1(x) & Z_2(x) & \cdots & Z_k(x) \\
Z_1^*(x^1) & Z_2^*(x^1) & \cdots & Z_k^*(x^1) \\
Z_1^*(x^2) & Z_2^*(x^2) & \cdots & Z_k^*(x^2) \\
\vdots & \vdots & \ddots & \vdots \\
Z_1^*(x^k) & Z_2^*(x^k) & \cdots & Z_k^*(x^k)
\end{bmatrix}
\]

Here \( x^1, x^2, \cdots, x^k \) are the ideal solutions of the objectives \( Z_1(x), Z_2(x), \cdots, Z_k(x) \) respectively. So \( U_r = \max \{ Z_r(x^1), Z_r(x^2), \cdots, Z_r(x^k) \} \) and \( L_r = \min \{ Z_r(x^1), Z_r(x^2), \cdots, Z_r(x^k) \} \).

Step 3: Using aspiration levels of each objective of the VMP (2.1.1) may be written as follows:

Find \( x \) so as to satisfy

\[
Z_r(x) \preceq L_r \quad (r = 1, 2, \cdots, k)
\]

\( x \in X \)
Here objective functions of (2.1.1) are considered as fuzzy constraints. This type of fuzzy constraints can be quantified by eliciting a corresponding membership function

\[
\mu_r(Z_r(x)) = \begin{cases} 
0 & \text{if } Z_r(x) \geq U_r \\
= d_r(x) & \text{if } L_r \leq Z_r(x) \leq U_r \ (r = 1, 2, \ldots, k) \\
= 1 & \text{if } Z_r(x) \leq L_r
\end{cases}
\]  

(2.2.2)

Here \(d_r(x)\) is a strictly monotonic decreasing function with respect to \(Z_r(x)\). Following figure illustrates the graph of the membership function \(\mu_r(Z_r(x))\).

![Membership function for minimization problem.](image)

Having elicited the membership functions (as in (2.2.2)), \(\mu_r(Z_r(x))\) for \(r=1,2,\ldots,k\), a general aggregation function

\[
\mu_D(x) = \mu_D\left(\mu_1(Z_1(x)), \mu_2(Z_2(x)), \ldots, \mu_k(Z_k(x))\right)
\]

is introduced. So a fuzzy multi-objective decision making problem can be defined as

\[
\text{Maximize } \mu_D(x) \quad (2.2.3)
\]

subject to

\[x \in X\]

Here one can adopt the well-known Bellman and Zadeh’s [1] fuzzy decision based on minimum operator (like Zimmermann’s approach [9]).

Then the problem (2.2.1) to be solved is reduced to
Maximize $\alpha$ \hfill (2.2.4)

Subject to $\mu_r(Z_r(x)) \geq \alpha \quad \text{for } r = 1, 2, \cdots, k$

$x \in X$

$0 \leq \alpha \leq 1.$

**Step 4:** Solve (2.2.4) to get a pareto optimal solution.

### 2.3. Pareto Optimality Test

A numerical test of pareto optimality for $x^*$ can be performed by solving the following problem:

Maximize $\sum_{r=1}^{k} \epsilon_r$ \hfill (2.3.1)

subject to $Z_r(x) + \epsilon_r = Z_r(x^*)$ \quad (for $r = 1, 2, \cdots, k$)

$x \in X$

Let $\bar{x}$ be an optimal solution to this problem (2.3.1). If $\epsilon_r = 0$, for all $r = 1, 2, \cdots, k$, then $x^*$ is a Pareto optimal solution. If at least one $\epsilon_r > 0$, (not $x^*$) $\bar{x}$ is a Pareto optimal solution of the MONLP (2.1.1).

### 2.4. Weights in Fuzzy Nonlinear Programming (FNLP)

Here, positive weights $W_r$ reflect the decision maker’s preferences regarding the relative importance of each objective goal $Z_r(x)$ for $r = 1, 2, 3, \cdots, k$. These weights can be normalized by taking $\sum W_r = 1$. To achieve the same objective function, suitable inverse weights are assigned to different membership functions in the fuzzy nonlinear programming (FNLP) method.

### 2.5. Weighted Fuzzy Non-linear Programming (WFNLP)

So introducing normalized weights in FNLP, (2.2.4) becomes

Maximize $\alpha$ \hfill (2.5.1)

subject to $w_r \mu_r(Z_r(x)) \geq \alpha \quad \text{for } r = 1, 2, \cdots, k$

$x \in X$

$0 \leq \alpha \leq 1$

where $\sum_{r=1}^{k} w_r = 1$. 
3. Fuzzy Programming Technique to Solve Multi-objective Portfolio Optimization Model

Vector Minimization Problem (VMP) form of (1.3) be

Minimize \[ -\ln(x) = \sum_{j=1}^{n} x_j \log x_j \] (1.4)

Minimize \[ -E_{r_1} (x) = -\sum_{j=1}^{n} r_j^1 x_j \]

Minimize \[ -E_{r_2} (x) = -\sum_{j=1}^{n} r_j^2 x_j \]

..............................................

Minimize \[ -E_{r_m} (x) = -\sum_{j=1}^{n} r_j^m x_j \]

Minimize \[ V_{r_1}(x) = \sum_{j=1}^{n} \sum_{i=1}^{n} \sigma_{ij}^1 x_j x_i \]

Minimize \[ V_{r_2}(x) = \sum_{j=1}^{n} \sum_{i=1}^{n} \sigma_{ij}^2 x_j x_i \]

..............................................

Minimize \[ V_{r_m}(x) = \sum_{j=1}^{n} \sum_{i=1}^{n} \sigma_{ij}^m x_j x_i \]

subject to

\[ \sum_{j=1}^{n} x_j = 1 \]

\[ x_j \geq 0, \ j = 1, 2, \ldots, n. \]

To solve VMP form (1.4), Step 1 of (2.2) is used. After that, according to Step 2, Pay-off matrix is formulated as follows:
Now we find, the upper bounds $U_{Er}^k$, $U_{Vr}^k$ and $U_{En}$ and the lower bounds $L_{Er}^k$, $L_{Vr}^k$ and $L_{En}$ where

$$U_{Er}^k = \max \{E_{rk}(x^1), E_{rk}(x^2), \ldots, E_{rk}(x^{2m+1})\} \quad \text{and} \quad L_{Er}^k = \min \{E_{rk}(x^1), E_{rk}(x^2), \ldots, E_{rk}(x^{2m+1})\}$$

$$U_{Vr}^k = \max \{V_{rk}(x^1), V_{rk}(x^2), \ldots, V_{rk}(x^{2m+1})\} \quad \text{and} \quad L_{Vr}^k = \min \{V_{rk}(x^1), V_{rk}(x^2), \ldots, V_{rk}(x^{2m+1})\}$$

(k=1, 2, \ldots, m) and $U_{En} = \max \{En(x^1), En(x^2), \ldots, En(x^{2m+1})\}$ and $L_{En} = \min \{En(x^1), En(x^2), \ldots, En(x^{2m+1})\}$. So $L_{Er}^k \leq E_{rk}(x) \leq U_{Er}^k$, $L_{Vr}^k \leq V_{rk}(x) \leq U_{Vr}^k$ and $L_{En} \leq En(x) \leq U_{En}$.

For simplicity the membership functions $\mu_{-Er}(\text{En}(x))$, $\mu_{Vr}(\text{En}(x))$, $\mu_{-Vr}(\text{En}(x))$ for the objective functions $E_{rk}(x)$, $V_{rk}(x)$ (k=1,2, \ldots, m), $En(x)$ respectively are defined as follows:

$$\mu_{-Er}(\text{En}(x)) = \begin{cases} 1 & \text{if } -E_{rk}(x) \leq -U_{Er}^k \\ \frac{-L_{Er}^k - (-E_{rk}(x))}{-L_{Er}^k - (-U_{Er}^k)} & \text{if } -U_{Er}^k < -E_{rk}(x) < -L_{Er}^k \\ 0 & \text{if } -E_{rk}(x) \geq -L_{Er}^k \end{cases}$$

$$\mu_{Vr}(\text{En}(x)) = \begin{cases} 1 & \text{if } V_{rk}(x) \leq U_{Vr}^k \\ \frac{U_{Vr}^k - V_{rk}(x)}{U_{Vr}^k - L_{Vr}^k} & \text{if } L_{Vr}^k < V_{rk}(x) < U_{Vr}^k \\ 0 & \text{if } V_{rk}(x) \geq U_{Vr}^k \end{cases}$$

and $\mu_{-En}(\text{En}(x)) = \begin{cases} 1 & \text{if } -En(x) \leq -U_{En} \\ \frac{-L_{En} - (-En(x))}{-L_{En} - (-U_{En})} & \text{if } -U_{En} \leq -En(x) < -L_{En} \\ 0 & \text{if } -En(x) \geq -L_{En} \end{cases}$
According to Step 3, having elicited the above membership functions crisp nonlinear programming problem of (1.4) is formulated as follows:

Maximize $\beta$
subject to

$$Er_k(x) \geq L_{Er_k}^k + \beta(U_{Er_k}^k - L_{Er_k}^k),$$
$$Vr_k(x) \leq U_{Vr_k}^k - \beta(U_{Vr_k}^k - L_{Vr_k}^k), \quad (k=1, 2, \cdots, m) \quad (3.1)$$
$$En(x) \geq L_{En} + \beta(U_{En} - L_{En}),$$
$$\sum_{j=1}^{n} x_j = 1,$$
$$x_j \geq 0, \quad j=1, 2, \cdots, n \quad \text{and} \quad \beta \in [0,1].$$

and similarly weighted FNLP of (1.4) is formulated as follows

Maximize $\beta$
subject to

$$Er_k(x) \geq L_{Er_k}^k + \beta(w_{ek}(U_{Er_k}^k - L_{Er_k}^k)),$$
$$Vr_k(x) \leq U_{Vr_k}^k - \beta(w_{ek}(U_{Vr_k}^k - L_{Vr_k}^k)), \quad (k=1, 2, \cdots, m) \quad (3.2)$$
$$En(x) \geq L_{En} + \beta(w(U_{En} - L_{En}),$$
$$\sum_{j=1}^{n} x_j = 1,$$
$$x_j \geq 0, \quad j=1, 2, \cdots, n,$$
$$w + \sum_{k=1}^{m} w_{ek} + \sum_{k=1}^{m} w_{vk} = 1 \quad \text{and} \quad \beta \in [0,1].$$
4. Numerical Examples

4.1. Numerical Examples (for Model-I & II)

Consider the three-security problems with expected returns vector and covariance matrix given by
\[
( \mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3 ) = (0.062, 0.146, 0.128)
\]
and
\[
\begin{bmatrix}
\sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \rho_{13}\sigma_1\sigma_3 \\
\rho_{12}\sigma_1\sigma_2 & \sigma_2^2 & \rho_{23}\sigma_2\sigma_3 \\
\rho_{13}\sigma_1\sigma_3 & \rho_{23}\sigma_2\sigma_3 & \sigma_3^2
\end{bmatrix} =
\begin{bmatrix}
0.0146 & 0.0187 & 0.0145 \\
0.0187 & 0.0854 & 0.0104 \\
0.0145 & 0.0104 & 0.0289
\end{bmatrix}
\]

Let \( \mathbf{x} = (x_1, x_2, x_3)^T \), where \( x_1, x_2, x_3 \) is the proportion of an asset invested in the 1-st, 2-nd and 3-rd security respectively.

So Model-I is
\[
\begin{align*}
\text{Maximize } & \mathbf{E}(\mathbf{x}) = 0.062 x_1 + 0.146 x_2 + 0.128 x_3 \\
\text{Minimize } & \mathbf{V}(\mathbf{x}) = 0.0146 x_1^2 + 0.0854 x_2^2 + 0.0289 x_3^2 \\
& + 2(0.0187 x_1 x_2 + 0.0145 x_1 x_3 + 0.0104 x_2 x_3)
\end{align*}
\]
subject to
\[
\begin{align*}
x_1 + x_2 + x_3 &= 1, \\
\text{and } & x_1, x_2, x_3 \geq 0.
\end{align*}
\]

and Model-II is
\[
\begin{align*}
\text{Maximize } & \mathbf{E}(\mathbf{x}) = - (x_1 \log x_1 + x_2 \log x_2 + x_3 \log x_3) \\
\text{Maximize } & \mathbf{E}(\mathbf{x}) = 0.062 x_1 + 0.146 x_2 + 0.128 x_3 \\
\text{Minimize } & \mathbf{V}(\mathbf{x}) = 0.0146 x_1^2 + 0.0854 x_2^2 + 0.0289 x_3^2 \\
& + 2(0.0187 x_1 x_2 + 0.0145 x_1 x_3 + 0.0104 x_2 x_3)
\end{align*}
\]
subject to
\[
\begin{align*}
x_1 + x_2 + x_3 &= 1, \\
\text{and } & x_1, x_2, x_3 \geq 0.
\end{align*}
\]

Pareto optimal solutions of Model-I and Model-II are
We see that model-I has one variable $x_i^*$ with zero value whereas there are all non-zero values of $x_i^*$, $x_j^*$, $x_k^*$ in Model-II. Here entropy is acted as a measure of dispersal of assets investment with small changes of $Er(x)$, $Vr(x)$. If an investor wishes to distribute his asset in various bonds, the PSPD (Model-II) will be more realistic for him.

For using different weights, optimal solution table are given below:

<table>
<thead>
<tr>
<th>Weights</th>
<th>$x_1^*$</th>
<th>$x_2^*$</th>
<th>$x_3^*$</th>
<th>$Er(x^*)$</th>
<th>$Vr(x^*)$</th>
<th>$En(x^*)$</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_e=1/3$</td>
<td>$w_v=1/3$</td>
<td>$w = 1/3$</td>
<td>0.21876</td>
<td>0.24989</td>
<td>0.53134</td>
<td>0.07828</td>
<td>0.038375</td>
</tr>
<tr>
<td>$w_e=0.47$</td>
<td>$w_v=0.06$</td>
<td>$w = 0.47$</td>
<td>0.26801</td>
<td>0.14770</td>
<td>0.58428</td>
<td>0.05049</td>
<td><strong>0.023983</strong></td>
</tr>
<tr>
<td>$w_e=0.09$</td>
<td>$w_v=0.44$</td>
<td>$w = 0.47$</td>
<td>0.79767</td>
<td>0.19756</td>
<td>0.00475</td>
<td><strong>0.09058</strong></td>
<td>0.04879</td>
</tr>
</tbody>
</table>

Here, results have been presented for model-II with the different weights to the objectives. The results follow a particular pattern. In type-II expected return is higher than that in type-I. In type-III risk is higher than that in type-I. Bold faces results give the better results of the respective objectives.
4.2. Numerical Example of GPSPD (i.e. Model-III):

Consider the three-security problems with expected returns vector and covariance matrix given by

\[(r_1^1, r_1^2, r_1^3) = (0.102, 0.046, 0.023)\]

\[
\begin{bmatrix}
\sigma_1^2 & \rho_{12}^2 \sigma_1^2 \sigma_2^2 & \rho_{13}^2 \sigma_1^2 \sigma_3^2 \\
\rho_{12}^2 \sigma_1^2 \sigma_2^2 & \sigma_2^2 & \rho_{23}^2 \sigma_2^2 \sigma_3^2 \\
\rho_{13}^2 \sigma_1^2 \sigma_3^2 & \rho_{23}^2 \sigma_2^2 \sigma_3^2 & \sigma_3^2
\end{bmatrix} = \begin{bmatrix}
0.0146 & 0.0187 & 0.0145 \\
0.0187 & 0.0854 & 0.0104 \\
0.0145 & 0.0104 & 0.0289
\end{bmatrix},
\]

\[(r_2^1, r_2^2, r_2^3) = (0.052, 0.176, 0.062)\]

\[
\begin{bmatrix}
\sigma_1^2 & \rho_{12}^2 \sigma_1^2 \sigma_2^2 & \rho_{13}^2 \sigma_1^2 \sigma_3^2 \\
\rho_{12}^2 \sigma_1^2 \sigma_2^2 & \sigma_2^2 & \rho_{23}^2 \sigma_2^2 \sigma_3^2 \\
\rho_{13}^2 \sigma_1^2 \sigma_3^2 & \rho_{23}^2 \sigma_2^2 \sigma_3^2 & \sigma_3^2
\end{bmatrix} = \begin{bmatrix}
0.0620 & 0.0146 & 0.0128 \\
0.0146 & 0.0854 & 0.0102 \\
0.0128 & 0.0102 & 0.0219
\end{bmatrix},
\]

\[(r_3^1, r_3^2, r_3^3) = (0.092, 0.044, 0.138)\]

\[
\begin{bmatrix}
\sigma_1^2 & \rho_{12}^3 \sigma_1^2 \sigma_2^3 & \rho_{13}^3 \sigma_1^2 \sigma_3^3 \\
\rho_{12}^3 \sigma_1^2 \sigma_2^3 & \sigma_2^3 & \rho_{23}^3 \sigma_2^3 \sigma_3^3 \\
\rho_{13}^3 \sigma_1^2 \sigma_3^3 & \rho_{23}^3 \sigma_2^3 \sigma_3^3 & \sigma_3^3
\end{bmatrix} = \begin{bmatrix}
0.0144 & 0.0137 & 0.0141 \\
0.0137 & 0.0824 & 0.0104 \\
0.0141 & 0.0104 & 0.0283
\end{bmatrix},
\]

For using different weights, optimal solution table are given below:

<table>
<thead>
<tr>
<th>Weights</th>
<th>Er_1(x*)</th>
<th>Er_2(x*)</th>
<th>Er_3(x*)</th>
<th>Vr_1(x*)</th>
<th>Vr_2(x*)</th>
<th>Vr_3(x*)</th>
<th>En(x*)</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w_{c1} = w_{c2} = w_{c3} = 1/7)</td>
<td>.06793</td>
<td>.09082</td>
<td>.08888</td>
<td>.03241</td>
<td>.02011</td>
<td>.02024</td>
<td>.77562</td>
<td>I</td>
</tr>
<tr>
<td>(w_{c1} = w_{c2} = w_{c3} = .03, w_{v1} = w_{v2} = w_{v3} = .43)</td>
<td>.07331</td>
<td>.09307</td>
<td>.08333</td>
<td>.03664</td>
<td>.02128</td>
<td>.02097</td>
<td>.68153</td>
<td>II</td>
</tr>
<tr>
<td>(w_{c1} = w_{c2} = .16, w_{c3} = .04, w_{v1} = w_{v2} = w_{v3} = .07, w = .43)</td>
<td>.05259</td>
<td>.06984</td>
<td>.11477</td>
<td>.02456</td>
<td>.01667</td>
<td>.01933</td>
<td>.58237</td>
<td>III</td>
</tr>
<tr>
<td>(w_{c1} = w_{c2} = w_{c3} = .1, w_{v1} = w_{v2} = .11, w_{v3} = .02, w = .43)</td>
<td>.08355</td>
<td>.07471</td>
<td>.08938</td>
<td>.04154</td>
<td>.01635</td>
<td>.01624</td>
<td>.40782</td>
<td>IV</td>
</tr>
</tbody>
</table>
Here, results have been presented for model-III with the different weights to the objectives. Here the results also follow a particular pattern (like table-II).

5. Conclusion

In this paper we consider a general applications of portfolio selection problem. Firstly, we consider a multi-objective Portfolio Selection based model and then added another entropy objective function, which is used by Shannon’s measure of entropy. Next this entropy-based problem is formulated in generalized form. Fuzzy non-linear programming technique is used to solve the problems. The models are illustrated with several numerical examples. Like multi-objective entropy based Portfolio Selection problem, entropy may be used in other fields of operations research and engineering sciences.

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References


